Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

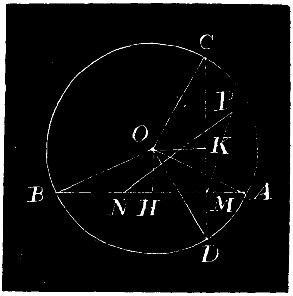
Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

Let M, N, P be the three random points, O the center of the circle, AB, the chord through M, N, and CD the chord through M perpendicular to AB. Draw OH and OK perpendicular to AB and CD.



Now N being between B and M, the angle M of the triangle MNP is obtuse if P lies in the segment CAD.

Let $OA = r$, $MN = x$, $\angle AOH = \theta$, $COK = \varphi$, and $\psi =$ the angle AB makes with some fixed line. Then $BH = r \sin\theta$, $OK = MH = r \sin\varphi$, area seg. $CAD = r^2(\varphi - \sin\varphi\cos\varphi)$; an element of the circle at M is $r\sin\theta\, d\theta\, r\sin\varphi\, d\varphi$, and at N it is $x\, dx\, d\psi$. The limits of θ are 0 and $\frac{1}{2}\pi$; of φ, $\frac{1}{2}\pi - \theta$ and $\frac{1}{2}\pi + \theta$; of x, 0 and $r(\sin\theta + \cos\varphi) = x'$, and doubled; and of ψ, 0 and 2π.

Hence, since the whole number of ways the three points can be taken is $\pi^3 r^6$, the chance that the angle M is obtuse is

$$p_1 = \frac{2}{\pi^3 r^6}\int_0^{\frac{1}{2}\pi}\int_{\frac{1}{2}\pi-\theta}^{\frac{1}{2}\pi+\theta}\int_0^{x'}\int_0^{2\pi} r^2(\varphi - \sin\varphi\cos\varphi)\, r\sin\theta\, d\theta\, r\sin\varphi\, d\varphi\, x\, dx\, d\psi$$

$$= \frac{4}{\pi^2 r^2}\int_0^{\frac{1}{2}\pi}\int_{\frac{1}{2}\pi-\theta}^{\frac{1}{2}\pi+\theta}\int_0^{x'} (\varphi - \sin\varphi\cos\varphi)\sin\theta\sin\varphi\, d\theta\, d\varphi\, x\, dx$$

$$= \frac{2}{\pi^2}\int_0^{\frac{1}{2}\pi}\int_{\frac{1}{2}\pi-\theta}^{\frac{1}{2}\pi+\theta} (\varphi - \sin\varphi\cos\varphi)(\sin\theta + \cos\varphi)^2 \sin\theta\cos\varphi\, d\theta\, d\varphi$$

$$= \frac{1}{3\pi^2}\int_0^{\frac{1}{2}\pi} (8\pi\sin^2\theta + 3\theta - 12\theta\sin^2\theta - 3\sin\theta\cos\theta - 6\sin^3\theta\cos\theta)\sin^2\theta\, d\theta$$

$= \frac{3}{8} - \frac{4}{3\pi^2}$. Hence the chance that the triangle is obtuse is $3p_1$ $= \frac{9}{8} - \frac{4}{\pi^2}$, and the chance that it is acute is $p = 1 - 3p_1 = \frac{4}{\pi^2} - \frac{1}{8}$.

DEMONSTRATION OF A PROPOSITION.

BY THE EDITOR.

THE following *Proposition* has been sent to us by a subscriber with the request that a demonstration be given.

As the proposition, in the form here given is new to us, and as we have obtained a very elementary demonstration, assuming that it may interest *other* subscribers, also, we submit it to our readers.

Proposition.—If any parallelogram be divided into two trapezoids by a straight line, and a straight line be drawn through the points of intersection of the diagonals of the two trapezoids thus formed, it will bisect the parallelogram.

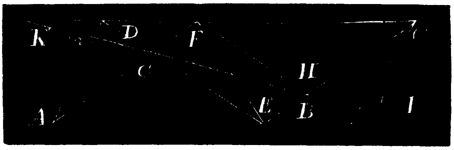
The truth of the above proposition may be inferred as a corollary depending upon the demonstration of the theorem of APOLLONIUS, viz.:

"If any quadrilateral be divided into two other quadrilaterals by a straight line, the intersections of the diagonals of the three quadrilaterals are on the same straight line."

For, when the quadrilateral is a parallelogram, the intersection of its diagonals is at its center; therefore the straight line joining the other two intersections passes through the center of the parallelogram, and therefore bisects it.

But, as the demonstration of the theorem of APOLLONIUS is somewhat complicated (see Am. Jour. of Science and Arts for 1857, Vol. 23, p. 224), we sought and obtained the following, more elementary, demonstration of the foregoing proposition.

Let *ABCD* represent any parallelogram; let it be divided into two trapezoids by the line *EF*, and draw the diagonals intersecting in *G*, *H*. Draw *GH* and prolong it in both directions to meet the sides produced in *I* and *K*.



Because of the parallel sides, and of the equal opposite angles at *G*, the triangles *AGE* and *FGD* are similar, as are, also, the triangles *EGI* and *DGK*; and because the sides *DG* and *GE* are common to the two pairs of similar triangles, we have

$$AE : EI :: FD : DK;$$

and, similarly,

$$EB : BI :: CF : FK.$$

Or, putting $AE = a$, $EB = b$, $CF = c$, $FD = d$, $BI = h$ and $DK = k$,

$$a : b+h :: d : k, \text{ whence } ak = bd + dh, \qquad (1)$$

$$c : d+k :: b : h, \quad \text{“} \quad ch = bd + bk. \qquad (2)$$

Subtracting (2) from (1) we have

$$ak - ch = dh - bk,$$

or, transposing *ch* and *bk*,

$$(a+b)k = (c+d)h.$$

But $a+b = c+d$; $\therefore h = k$; and because the angles at *I* and *K* are alternate angles they are equal, and the triangles *BIn* and *DKm* are identical; therefore $Am = Cn$, and consequently *IK* bisects the parallelogram.